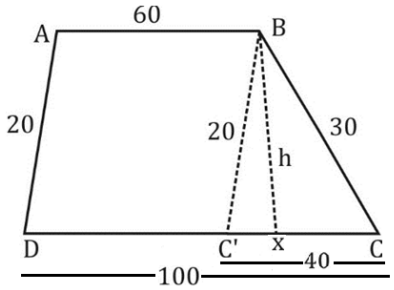
Quant Quiz Advance Level for SSC 22th December (Solutions)

S1. Ans.(b)

Sol.



Ara of $\Delta BCC' = \sqrt{s(s-a)(s-b)(s-c)}$

$=\sqrt{45(25)(15)(5)}$

$= 3 \times 5 \times 5\sqrt{15}$

$= 75\sqrt{15}$

Area of $\Delta BCC' = \frac{1}{2} \times CC' \times BX$

$75\sqrt{15} = \frac{1}{2} \times 40 \times h$

$\frac{75\sqrt{15}}{20} = h$

$h = \frac{15}{4}\sqrt{15}$

Area of trapezium $= \frac{1}{2}$(sum of parallel sides) $\times$ height

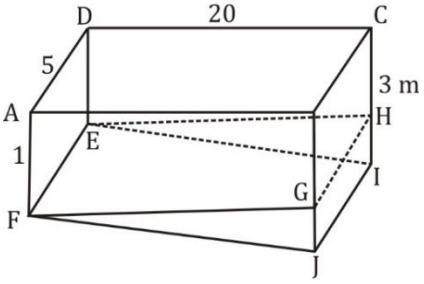
$= \frac{1}{2} \times 160 \times \frac{15}{4}\sqrt{15}$

$= 20 \times 15\sqrt{15}$

$= 300\sqrt{15}$

S2. Ans.(d)

Sol.



Area of cuboid ABCDEFGH $= 1 \times 5 \times 20 = 100\ cm^3$

volume of triangular prism = Area of triangle × depth

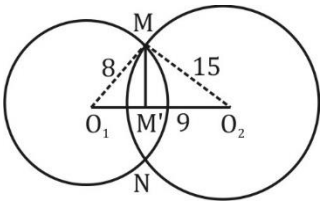
$= \frac{1}{2} \times 20 \times 2 \times 5$

$= 100$

Total volume $= 200\ m^3$

S3. Ans.(b)

Sol.



$AB = r_1 + r_2 - O_1O_2$

$= 23 - 17$

$= 6cm$

$\Rightarrow$ Area of ΔO_1MO_2

$\frac{1}{2} O_1M \times MO_2 = \frac{1}{2} O_1O_2 \times MM^1$

$\frac{1}{2} \times 8 \times 15 = \frac{1}{2} 17 \times MM^1$

$\frac{120}{17} = MM^1$

$MN = 2 \times \frac{120}{17} = \frac{240}{17} cm$

$AB = 6\ cm$

Area of rectangle ABCD = AB × BC

$= 6 \times \frac{240}{17}$

$= \frac{1440}{17}\ cm^2$

S4. Ans.(a)

Sol. $\sin (90 + 15°) = \cos 15°$

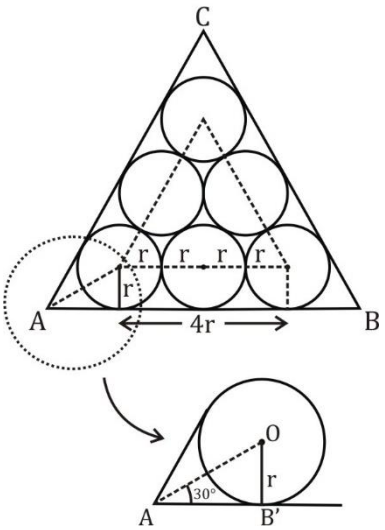
$= \cos (45° - 30°)$

$= \cos 45°. \cos 30° + \sin 45° \sin 30°$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$

$= \frac{\sqrt{6}+\sqrt{2}}{4}$

S5. Ans.(c)
Sol.



In Δ AOB'

$\tan 30° = \frac{OB}{AB'}$

$AB' = r\sqrt{3}$

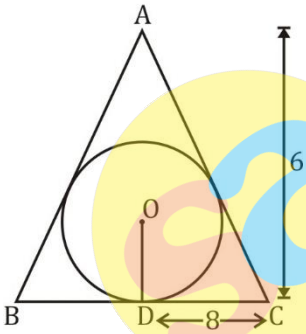
Length of $AB = \sqrt{3}r + 4r + \sqrt{3}r$

$= 2\sqrt{3}r + 4r$

$= (2\sqrt{3} + 4)\, r$

Perimeter of triangle $= 3 \times (2\sqrt{3} + 4)r$

S6. Ans.(a)
Sol.



Sphere inside the cone is equivalent to the circle inside the triangle

Slant height of cone = AC

$AC = \sqrt{DA^2 + DC^2}$

$= 10$ cm

In radius of the triangle $= \frac{\text{Area}}{S}$

$= \frac{\frac{1}{2} \times 16 \times 6}{18}$

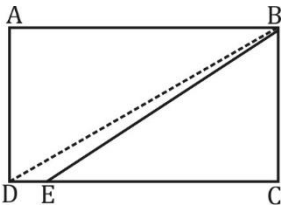
$= \frac{8}{3}$ cm

Vol. of the sphere $= \frac{4}{3}\pi \times \left(\frac{8}{3}\right)^3$

$= 8\pi \left(\frac{4}{3}\right)^4 \text{cm}^3$

S7. Ans.(b)

Sol.



Area of Δ BEC $= \frac{8}{9}$ Area of ΔBDC

& Area of ΔADB = Area of ΔBCD

Hence

Area of ABCD $= \frac{18}{9}$ area of ΔBDC

$$\frac{\text{Area of } \Delta BEC}{\text{Area of ABCD}} = \frac{8/9}{18/9}$$

Area of ΔBEC $= \frac{4}{9}$ Area of ABCD

S8. Ans.(a)

Sol.

$$\frac{AC}{\sin 30^\circ} = \frac{10}{\sin 75^\circ}$$

$\Rightarrow AC = \frac{5}{\sin(45+30)}$ & $AB = \frac{AC}{\sqrt{3}}$

SO

$$AB = \frac{5}{\sqrt{3}\frac{(\sqrt{3}+1)}{2\sqrt{2}}} = \frac{10\sqrt{2}}{3+\sqrt{3}}$$

S9. Ans.(c)

Sol.

$\frac{1}{1} - 2\cot^2 58^\circ + 2\,\text{cosec}^2\, 58^\circ - 4$

$1 + 2 - 4 = -1$

S10. Ans.(c)

Sol. Let $\theta = 45^\circ$

$\frac{x}{2\sqrt{2}} + \frac{y}{2\sqrt{2}} = \frac{1}{2}$ & $x = y$

$\Rightarrow x + y = \sqrt{2}$

$\Rightarrow x = y = \frac{1}{\sqrt{2}}$

$\therefore x^2 + y^2 = 1$

S11. Ans(d)

Sol. $x^2 + px + q = 0$ having roots as $\tan 30°$ & $\tan 15°$

Sum of roots,

$\tan 30° + \tan 15° = -P$

$$\frac{1}{\sqrt{3}} + \tan(45° - 30°) = -p$$

$$\Rightarrow \frac{1}{\sqrt{3}} + \frac{1 - 1/\sqrt{3}}{1 + 1/\sqrt{3}} = -p$$

$$\Rightarrow \frac{1}{\sqrt{3}} + \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = -p$$

$$\Rightarrow \frac{\sqrt{3} + 1 + 3 - \sqrt{3}}{\sqrt{3}(\sqrt{3} + 1)} = -p$$

$$\Rightarrow \frac{4}{(\sqrt{3} + 3)} = -p$$

Product of roots,

$$\& \frac{1}{\sqrt{3}} . \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = q$$

$$\Rightarrow \frac{\sqrt{3} - 1}{3 + \sqrt{3}} = q$$

$$\Rightarrow 2 + q - p = \frac{4}{3 + \sqrt{3}} + 2 + \frac{\sqrt{3} - 1}{3 + \sqrt{3}}$$

$$\Rightarrow \frac{3 + \sqrt{3}}{3 + \sqrt{3}} + 2 = 3$$

S12. Ans.(d)

Sol.

$ab - a^2 = b^2 \Rightarrow a^2 + b^2 - ab = 0$

$\Rightarrow a^3 + b^3 = 0$

S13. Ans.(c)

Sol.

Vol. of wire = Vol. of the sphere

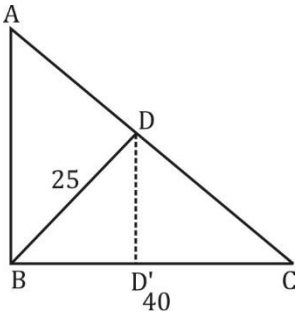
$$\pi \left(\frac{3}{10}\right)^2 \ell = \frac{4}{3}\pi \times 729$$

$\ell = 400 \times 27$

$\ell = 10800$ cm

S14. Ans.(d)

Sol.



Shortest median will be drawn on to the biggest side AC.

BD = 25 cm

& Median of right angled triangle = Circumradi of triangle

Hence AD = DC = BD = 25 cm

$DD' = \sqrt{BD^2 - BD'^2} = 15cm$

Hence Area of $\Delta BDC = \frac{1}{2} \times 40 \times 15$

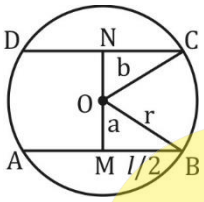
$= 300$ sq. units

Area of $\Delta ABC = 2 \times$ Area of ΔBDC

$= 600$ sq. units

S15. Ans.(d)

Sol.



Let the radii of the circle is r cm

From ΔOMB,

$r^2 = a^2 + \left(\frac{\ell}{2}\right)^2$

$r^2 = \frac{4a^2 + \ell^2}{4}$

From ΔONC,

$NC^2 = OC^2 - ON^2$

$= \frac{4a^2 + \ell^2}{4} - b^2$

$= \frac{4a^2 - 4b^2 + \ell^2}{4}$

$NC = \frac{\sqrt{4a^2 - 4b^2 + \ell^2}}{2}$

$CD = 2NC = \sqrt{4a^2 - 4b^2 + \ell^2}$

S16. Ans.(a)

Sol.

Let $a + b + c = p$

$$\frac{x - a^2}{p - a} + \frac{x - b^2}{p - b} + \frac{x - c^2}{p - c} = 3p + a + b + c$$

$$\frac{(x - a^2)}{p - a} - (p + a) + \frac{(x - b^2)}{p - b} - (p + b) + \frac{(x - c^2)}{p - c} - (p + c) = 0$$

$$\frac{(x - a^2) - p^2 + a^2}{p - a} + \frac{(x - b^2) - p^2 + b^2}{p - b} + \frac{(x - c^2) - p^2 + c^2}{p - c} = 0$$

$$(x - p^2)\left[\frac{1}{b + c} + \frac{1}{c + a} + \frac{1}{a + b}\right] = 0$$

$\Rightarrow x = p^2 = (a + b + c)^2$

S17. Ans.(a)

Sol.

Let $p = 2, q = -2, r = 1$

$\therefore \frac{1}{p} + \frac{1}{q} + \frac{1}{r} = 1$

S18. Ans.(b)

Sol.

$$z + \frac{1}{z} = 1$$

$\Rightarrow z^2 - z + 1 = 0$

$\Rightarrow (z + 1)(z^2 - z + 1) = 0 \qquad (\text{if } z \neq -1)$

$\Rightarrow z^3 + 1 = 0$

Or $z^3 = -1$ & $z \neq -1$

Now, $z^{64} + \frac{1}{z^{64}} = z^{63}z + \frac{1}{z^{63}.z}$

$$\Rightarrow (z^3)^{21}.z + \frac{1}{(z^3)^{21}.z} = -z - \frac{1}{z}$$

$= -1$

S19. Ans.(a)

Sol.

$$2\left[a^{\frac{1}{3}} + \frac{1}{a^{\frac{1}{3}}}\right] = 5$$

$$2a^{\frac{2}{3}} - 5a^{\frac{1}{3}} + 2 = 0$$

$$\Rightarrow \left(a^{\frac{1}{3}} - 2\right)\left(2a^{\frac{1}{3}} - 1\right) = 0$$

$$\text{or, } a^{\frac{1}{3}} = 2, a^{\frac{1}{3}} = \frac{1}{2}$$

$$\text{or, } a = 8, a = \frac{1}{8}$$

S20. Ans.(a)

Sol. For Max vol. of a cylinder of radius r & height h inside a sphere of Radius 'R'

$r = \sqrt{\frac{2}{3}}R$ & $h = \frac{2R}{\sqrt{3}}$ [Remember this]

&

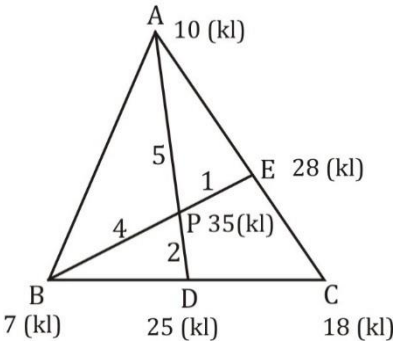
$\therefore V_{max} = \pi r^2 h = \pi \times \frac{2}{3}R^2 \times \frac{2R}{\sqrt{3}}$

$= \frac{4 \times R^3}{3\sqrt{3}}$ [Remember this]

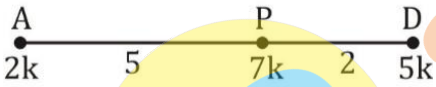
$\therefore = \frac{4\pi}{3\sqrt{3}} \times \left(\frac{7}{2}\right)^3 = \frac{343\pi}{6\sqrt{3}}$.

S21. Ans.(d)

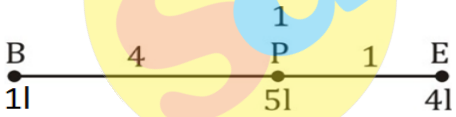
Sol.



By applying weight balance method, on side APD



Similarly on side BPE



L.C.M. of 5l& 7K $\Rightarrow$ 35 kl

Hence weight on P = 35 kl

Now weight on A = 10 kl

Now weight on D = 25 kl

Now weight on B = 7 kl

Now weight on E = 28 kl

Hence

Weight on B + weight on C= weight on D

7 kl + weight on C = 25 kl

= 18 kl

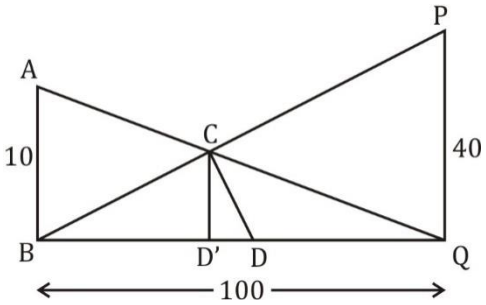
Now from side AEC

$x \times 10\ kl = y \times 18\ kl$

$\frac{x}{y} = \frac{9}{5}$

S22. Ans.(a)

Sol.



∵ CD is the median hence Area will be divided into 2 equal parts.

Area of ΔBCD = ½ Area of ΔBCQ

Area of BCQ = ½ BQ × CD'

$\frac{1}{CD'} = \frac{1}{PQ} + \frac{1}{AB}$

$= \frac{1}{10} + \frac{1}{40} \Rightarrow \frac{1}{8}$

CD'= 8

$\text{Area of } \Delta BCD = \frac{1}{2} \times 100 \times 8$

$= 400$

Area of ΔBCD = 200 cm^2

S23. Ans.(d)

Sol.

$\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{a^2\beta^2}$

now, $\alpha + \beta = -\frac{b}{a} \& \alpha\beta = \frac{c}{a}$

$\Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = \frac{b^2}{a^2}$

$\Rightarrow \alpha^2 + \beta^2 = \frac{b^2}{a^2} - \frac{2c}{a}$

$\therefore \ -\frac{b}{a} = \frac{b^2 - 2ac}{a^2.\frac{c^2}{a^2}} = \frac{b^2 - 2ac}{c^2}$

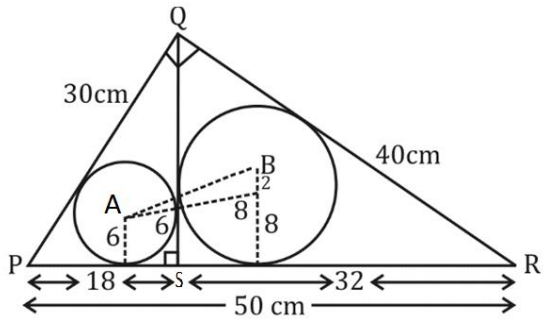
$\text{or}, -\frac{b}{a} = \frac{b^2}{c^2} - \frac{2a}{c}$

$\text{or}, b^2a + bc^2 = 2a^2c$

$\text{or}, \frac{b^2}{ac} + \frac{bc}{a^2} = 2$

S24. Ans.(d)

Sol.



In right angle ΔPQR

$PR = \sqrt{30^2 + 40^2} = 50$ cm

Area of Δ PQR $= \frac{1}{2} \times 30 \times 40 = \frac{1}{2} \times QS \times 50$

$\Rightarrow QS = 24$ cm

Again in right angled ΔQSP

$PS = \sqrt{30^2 - 24^2} = 18$ cm

And $SR = \sqrt{40^2 - 24^2} = 32$ cm

⇒ Inradius of Δ PQS

$r_1 = \frac{P+B-H}{2} = \frac{18+24-30}{2} = 6$cm

And In radius of ΔQSR

$r_2 = \frac{P+B-H}{2} = \frac{24+32-40}{2} = 8$ cm

$\therefore AB = \sqrt{14^2 + 2^2} = \sqrt{200}$

S25. Ans.(d)

Sol. cot (90° + 33°) cot (90° + 43°) cot (180° – 33°) cot (180° – 43°)

⇒ (–tan 33°).(–tan 43°).(-cot 33°).(-cot 43°) = 1