# Quant Mega Quiz for SSC CGL Tier – 2 (Solutions)

**S1. Ans.(d)**

**Sol.** For maximum value

$$a = b = c = d = \frac{1}{4}$$

$$(1+a)(1+b)(1+c)(1+d)$$

$$= \left(1+\frac{1}{4}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{4}\right)$$

$$= \left(\frac{5}{4}\right)^4$$

**S2. Ans.(a)**

**Sol.**

$$\frac{1}{\sqrt[3]{4}+\sqrt[3]{2}+1} = a\sqrt[3]{4} + b\sqrt[3]{2} + c$$

$$\Rightarrow \frac{1}{2^{\frac{2}{3}} + 2^{\frac{1}{3}} + 1} = a(2)^{\frac{2}{3}} + b(2)^{\frac{1}{3}} + c$$

On multiplying numerator and denominator by $\left(2^{\frac{1}{3}} - 1\right)$

$$\Rightarrow \frac{2^{\frac{1}{3}} - 1}{\left(2^{\frac{1}{3}} - 1\right)\left(2^{\frac{2}{3}} + 2^{\frac{1}{3}} + 1\right)} = a.2^{\frac{2}{3}} + b.2^{\frac{1}{3}} + c$$

$$\frac{2^{\frac{1}{3}} - 1}{2 - 1} = a.2^{\frac{2}{3}} + b.2^{\frac{1}{3}} + c$$

$$\therefore a = 0, b = 1, c = -1$$

$$a + b + c = 0 + 1 - 1 = 0$$

**S3. Ans.(b)**

**Sol.**

$$a = \frac{\sqrt{5}+1}{\sqrt{5}-1} = \frac{\sqrt{5}+1}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}$$

$$= \frac{\left(\sqrt{5}+1\right)^2}{5-1} = \frac{5+1+2\sqrt{5}}{4}$$

$$a = \frac{3+\sqrt{5}}{2}$$

*Similarly,*

$$b = \frac{3-\sqrt{5}}{2}$$

$$a + b = \frac{3+\sqrt{5}}{2} + \frac{3-\sqrt{5}}{2} = 3$$

and $ab = \frac{\sqrt{5}+1}{\sqrt{5}-1} \times \frac{\sqrt{5}-1}{\sqrt{5}+1} = 1$

$$\frac{a^2+ab+b^2}{a^2-ab+b^2}=\frac{(a+b)^2-ab}{(a+b)^2-3ab}$$
$$=\frac{3^2-1}{3^2-3}=\frac{9-1}{9-3}=\frac{8}{6}=\frac{4}{3}$$

**S4. Ans.(c)**

**Sol.**

$$x=(\sqrt[3]{a+\sqrt{a^2+b^3}}+\sqrt[3]{a-\sqrt{a^2+b^3}}$$

On cubing both sides.

$$x^3=\left(\sqrt[3]{a+\sqrt{a^2+b^3}}+\sqrt[3]{a-\sqrt{a^2+b^3}}\right)^3$$
$$=\left(\sqrt[3]{a+\sqrt{a^2+b^3}}\right)^3+\left(\sqrt[3]{a-\sqrt{a^2+b^3}}\right)^3+3\left(\sqrt[3]{a+\sqrt{a^2+b^3}}\right)\left(\sqrt[3]{a-\sqrt{a^2+b^3}}\right)$$
$$\left[\sqrt[3]{a+\sqrt{a^2+b^3}}+\sqrt[3]{a-\sqrt{a^2+b^3}}\right]$$
$$=a+\sqrt{a^2+b^3}+a-\sqrt{a^2+b^3}+3\left[\left(a+\sqrt{a^2+b^3}\right)\left(a-\sqrt{a^2+b^3}\right)\right]^{\frac{1}{3}}x$$
$$=2a+3\,(a^2-a^2-b^3)^{\frac{1}{3}}x$$
$$\therefore x^3=2a+3\,(-b)x$$
$$x^3+3bx=2a$$

**S5. Ans.(c)**

**Sol.**

Average of $x$ and $\frac{1}{x}$ = M

$$\Rightarrow \frac{x+\frac{1}{x}}{2}=\text{M}, x+\frac{1}{x}=2\text{M}$$

On squaring both sides

$$\left(x+\frac{1}{x}\right)^2=(2\text{M})^2$$
$$x^2+\frac{1}{x^2}+2=4\text{M}^2$$
$$\Rightarrow x^2+\frac{1}{x^2}=4\text{M}^2-2$$

$$\text{Average } x^2+\frac{1}{x^2}=\frac{x^2+\frac{1}{x^2}}{2}$$
$$=\frac{4\text{M}^2-2}{2}=\frac{2(2\text{M}^2-1)}{2},=2\text{M}^2-1$$

**S6. Ans.(d)**

**Sol.**

$a+b=\frac{1}{2+\sqrt{3}}+\frac{1}{2-\sqrt{3}}=4$ and,

$\text{ab}=\frac{1}{2+\sqrt{3}}\times\frac{1}{2-\sqrt{3}}=1$

$$\Rightarrow \frac{1}{a}+\frac{1}{b}=\frac{a+b}{ab}=4$$

S7. Ans.(a)

Sol.

$(a^2 + 4b^2 + 4b - 4ab - 2a - 8)$
$= (a - 2b)^2 - 2(a - 2b) - 8$
$= x^2 - 2x - 8$ where $(a - 2b) = x$
$= x^2 - 4x + 2x - 8$
$= x(x - 4) + 2(x - 4)$
$= (x + 2)(x - 4)$
$= (a - 2b - 4)(a - 2b + 2)$


S8. Ans.(b)

Sol.

$a^3 - b^3 = 56$ ....(i)
$a - b = 2$
$\because a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$
$56 = 2\,(a^2 + b^2 + ab)$
$28 = [(a - b)^2 + 3ab]$
$ab = 8$
$(a - b)^2 = a^2 + b^2 - 2ab$
$a^2 + b^2 = 20$

S9. Ans.(b)

Sol.

$a = \sqrt{6} + \sqrt{5}$
$\Rightarrow a^2 = 11 + 2\sqrt{30}$ & $b^2 = 11 - 2\sqrt{30}$
& ab = 1
$\Rightarrow 2a^2 - 5ab + 2b^2$
$\Rightarrow 22 + 4\sqrt{30} - 5 + 22 - 4\sqrt{30}$
$= 44 - 5 = 39$

S10. Ans.(d)

Sol.

$x = 3 + 2\sqrt{2}$
$\Rightarrow \frac{1}{x} = 3 - 2\sqrt{2}$
& $x + \frac{1}{x} = 6$ & $x - \frac{1}{x} = 4\sqrt{2}$
$\Rightarrow x^3 + \frac{1}{x^3} = 198$ & $x^3 - \frac{1}{x^3} = 140\sqrt{2}$

S11. Ans.(c)

Sol.

$a^2 + b^2 + \frac{1}{a^2} + \frac{1}{b^2} = 4$
Put a = b = 1
Thus, $a^2 + b^2 = 2$

**S12. Ans.(d)**
**Sol.**

$x+\frac{1}{x}=\sqrt{3}$

$\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3.x.\frac{1}{x}\left(x+\frac{1}{x}\right)$

$3\sqrt{3}=x^3+\frac{1}{x^3}+3\times\sqrt{3}$

$x^3+\frac{1}{x^3}=0$

**S13. Ans.(c)**
**Sol.**

$x^3+3x^2+3x=7$

$x^3+3.x^2.1+3.1^2.x+1^3-1^3=7$

$(x+1)^3-1=7$

$(x+1)^3=8$

$x+1=\pm2$

$x=1$

**S14. Ans.(b)**
**Sol.**

$2x+\frac{2}{x}=1$

$x+\frac{1}{x}=\frac{1}{2}$

$x^3+\frac{1}{x^3}=(Value)^2-3(Value)$

$=\frac{1}{8}-3\times\frac{1}{2}$

$=\frac{-11}{8}$

**S15. Ans.(c)**
**Sol.**

$2x+\frac{1}{3x}=6$

Multiply by $\frac{3}{2}$ both sides,

$3x+\frac{1}{2x}=9$

**S16. Ans.(a)**
**Sol.**

$x=\left(\sqrt{2}-1\right)^{-\frac{1}{2}}$

$=\frac{y}{\sqrt{\sqrt{2}-1}}$

$x^2=\frac{1}{\sqrt{2}-1}=\left(\sqrt{2}+1\right)$

$\frac{1}{x^2}=\left(\sqrt{2}-1\right)$

$x^2-\frac{1}{x^2}=\left(\sqrt{2}+1\right)-\left(\sqrt{2}-1\right)$

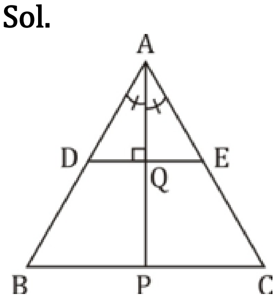
$=2$

**S17. Ans.(d)**
**Sol.**
$x^2 + y^2 - 2x + 6y + 10 = 0$
$(x^2 - 2x + 1) + (y^2 + 2.y.3 + 9) = 0$
$(x-1)^2 + (y+3)^2 = 0$
x = 1, y = -3
$x^2 + y^2 = 1 + 9$
= 10

**S18. Ans.(a)**
**Sol.**
$x^{\frac{1}{3}} + y^{\frac{1}{3}} = z^{\frac{1}{3}}$
Cubing both sides,
$x + y + 3\left(x^{\frac{1}{3}} + y^{\frac{1}{3}}\right)(xy)^{\frac{1}{3}} = z$
$x + y + 3(z)^{\frac{1}{3}}(xy)^{\frac{1}{3}} = z$
$(z + y - z) = -3(xyz)^{\frac{1}{3}}$
Again cubing both sides,
$(x + y - z)^3 = -27xyz$
$(x + y - z)^3 + 27xyz = 0$

**S19. Ans.(d)**
**Sol.**
$a^2 = 2$, then (a + 1)
$a = \sqrt{2}$
$(a + 1) = (\sqrt{2} + 1)$
Now, check from option, (d)
$\frac{a-1}{3-2a} = \frac{(\sqrt{2}-1)}{3-2(\sqrt{2})} = \frac{(\sqrt{2}-1)}{(3-2\sqrt{2})}\frac{(3+2\sqrt{2})}{(3+2\sqrt{2})}$
$= 3\sqrt{2} + 4 - 3 - 2\sqrt{2}$
$= (\sqrt{2} + 1)$

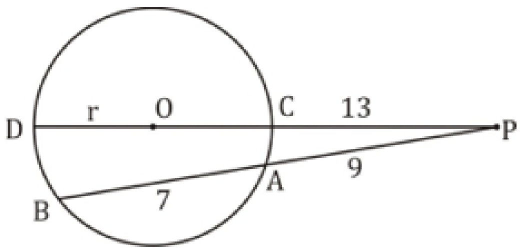
**S20. Ans.(a)**
**Sol.**
If $x + \frac{1}{x} = \sqrt{3}$, $x^6 = -1$ & power difference of 6 = 0
$\underbrace{x^{206} + x^{200}}_{0} + \underbrace{x^{90} + x^{84}}_{0} + \underbrace{x^{18} + x^{12}}_{0} + \underbrace{x^{6} + 1}_{0}$
= 0

**S21. Ans.(b)**
**Sol.**


∠DAQ = ∠EAQ
∠AQD = ∠AQE = 90°
AQ is common in ΔAQD and ΔAQE.
∴ Δ AQD ≅ ΔAQE
∴ AD = AE

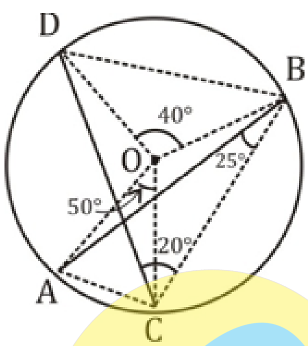


**S22. Ans.(d)**
**Sol.**




Let centre of circle is 'O' and
Radius = r cm
PC = 13 – r, PD = 13 + r
PC × PD = PA × PB
$\Rightarrow (13 - r)(13 + r) = 9 \times 16$
$\Rightarrow r = 5$ cm

**S23. Ans.(a)**
**Sol.**




∠AOC = 50°
∴ ∠ABC = 25°
And $\angle BOC = 40°$
∴ $\angle BCD = 20°$
$\angle BPC = 180° - \angle PBC - \angle BCP$
$\angle BPC = 180° - 25° - 20°$
$\angle BPC = 135°$
∴ ∠BPD = 180° - 135°
∠BPD = 45°

**S24. Ans.(c)**
**Sol.**
∠ADB= 90° (∵ angle in half circle)
ABCD is a cyclic quadrilateral
∴ ∠ BAD = 180 – ∠BCD
⇒ ∠BAD = 180 – 130° = 50°
∠ABD =180°–(∠ADB + ∠BAD)
∠ABD = 180–(90° + 50°)
∠ABD = 40°

**S25. Ans.(c)**

**Sol.**

Common tangent of two touching circle = $\sqrt{4Rr}$

(where R and r radius of both circle)

$\sqrt{4R_1R_2} = \sqrt{4R_1R_3} + \sqrt{4R_2R_3}$

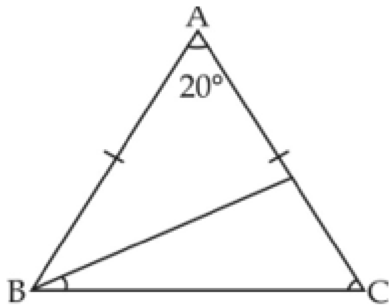
$\therefore \sqrt{4 \times 9 \times 4} = \sqrt{4 \times 9 \times R_3} + \sqrt{4 \times R_3 \times 4}$

$12 = \sqrt{R_3}[6 + 4]$

$R_3 = 1.44$ cm

**S26. Ans.(c)**

**Sol.**




$\because$ AB = AC

$\therefore \angle ABC = \angle ACB$

$\angle ABC + \angle ACB = 180° - 20°$

$\therefore \angle ABC = \angle ACB = 80°$

Use Sine rule:

In Δ ADB,

$\frac{\sin 20°}{BD} = \frac{\sin \angle ABD}{AD}$

$\Rightarrow BD = \frac{AD \sin 20°}{\sin \angle ABD}$ ...(i)

In ΔBDC,

$\frac{\sin \angle BDC}{BC} = \frac{\sin 80°}{BD}$

$\Rightarrow BD = \frac{BC \sin 80°}{\sin \angle BDC}$ ...(ii)

From (i) and (ii) we can say

$\frac{AD \sin 20°}{\sin \angle ABD} = \frac{BC \sin 80°}{\sin \angle BDC}$

$\angle BDC = \angle ABD + \angle DAB$

Let $\angle ABD = x°$

$\frac{\sin 20°}{\sin x°} = \frac{\sin 80°}{\sin(20+x)°}$ ($\because$AD=BC)

$\frac{\sin(20+x)}{\sin x} = \frac{\sin 80°}{\sin 20°}$

$\sin 20° \cot x + \cos 20° = 4 \cos 20° \cos 40°$

$\sin 20° \cot x + \cos 20° = 2 \cos 60° + 2 \cos 20°$

$\sin 20° \cot x = 1 + \cos 20°$

$\cot x = \frac{2 \cos^2 10°}{2 \sin 10° \cos 10°}$

$\cot x = \cot 10°$

$x = 10°$

$\therefore \angle DBC = 80° - 10° = 70°$

**S27. Ans.(c)**

**Sol.**

∠ADP = 100° (Given)
∴ ∠PDE = 80°
∠DPE = 90° (Angle of half circle)
∴ ∠PED = 180 – (∠DPE + ∠PDE)
∠PED = 10°
In same way
$\angle QDE = 10°$
∴ Consider Δ DRE
∠DRE = 180° – (∠RDO + ∠REO)
∠DRE = 180° – (10° + 10°)
∠DRE = 160°
∴ ∠PRD = 180° – ∠DRE
∠PRD = 180° – 160° = 20°

**S28. Ans.(d)**

**Sol.**

$\angle PCB = 180 - \angle BAP$
$\angle PCB = 180 - 60$
$\angle PCB = 120°$

**S29. Ans.(c)**

**Sol.**

Smaller circle passes through 'O' and touches outer circle at P
∴ OP will be a diameter of smaller circle
Then ∠ORP = 90° (angle of half circle)
So we can say OR ⊥ SP
∴ SR = PR (∵ SP is chord of outer circle)
$SR = \sqrt{OS^2 - OR^2}$
$SR = \sqrt{5^2 - 4^2}$
$SR = 3\ cm$
$SP = 3\ cm$

**S30. Ans.(a)**

**Sol.**

AC and BD are two chords intersecting at 'O'
OA × OC = OD × OB
And AC is angle bisector of ∠DAB
∴ AD : AB = OD : OB
∵ AD = AB
∴ OD = OB
OD × OB = OA × OC
$OA = \frac{OD^2}{OC} = \frac{8\times8}{4} \Rightarrow OA = 16$
$\therefore AC = 16 + 4 = 20\ cm$