Quant Mega Quiz for SSC Tier – 1 (Solutions)

S1. Ans.(a)

Sol.

$x + \frac{1}{x} = \sqrt{3}$

$x^3 + \frac{1}{x^3} = 0$

$x^6 + 1 = 0$

$x^{24} + x^{18} + x^{12} + x^6$

$= x^{18}(x^6 + 1) + x^6(x^6 + 1)$

$= x^{18} \times 0 + x^6 \times 0$

$= 0 + 0 = 0$

S2. Ans.(d)

Sol.

$x = 1 + \sqrt{2} + \sqrt{3}$

$x - 1 = \sqrt{2} + \sqrt{3}$

Squaring both sides

$x^2 + 1 - 2x = 2 + 3 + 2\sqrt{6}$

$x^2 + 1 - 2x = 5 + 2\sqrt{6}$

$x^2 - 2x - 4 = 2\sqrt{6}$

$x^2 - (2x + 4) = 2\sqrt{6}$

Squaring both sides

$[x^2 - (2x + 4)]^2 = 4 \times 6$

$x^4 + 4x^2 + 16 + 16x - 2x^2(2x + 4) = 24$

$x^4 + 4x^2 + 16x - 4x^3 - 8x^2 = 8$

$x^4 - 4x^3 - 4x^2 + 16x = 8$

$2x^4 - 8x^3 - 8x^2 + 32x = 16$

$2x^4 - 8x^3 - 5x^2 - 3x^2 + 26x + 6x = 16$

$2x^4 - 8x^3 - 5x^2 + 26x - 3x^2 + 6x = 16$

$2x^4 - 8x^3 - 5x^2 + 26x - 3(x^2 - 2x) = 16$

$2x^4 - 8x^3 - 5x^2 + 26x - 3(4 + 2\sqrt{6}) = 16$

$2x^4 - 8x - 5x^2 + 26x - 28 = 28 + 6\sqrt{6} - 28 = 6\sqrt{6}$

S3. Ans.(b)
Sol.
$$\left(\frac{x^2+y^2-2xy}{x^2+y^2}\right) \div \left(\frac{x^3-y^3-3xy(x-y)}{x-y}\right)$$
$$\Rightarrow \frac{(x-y)^2}{x^2+y^2} \div \frac{(x-y)^3}{(x-y)}$$
$$= \frac{(x-y)^2}{x^2+y^2} \times \frac{1}{(x-y)^2}$$
$$= \frac{1}{x^2+y^2}$$

S4. Ans.(a)
Sol.
a + b + c = 0
$$\frac{1}{(a+b)(b+c)} + \frac{1}{(b+c)(c+a)} + \frac{1}{(c+a)(a+b)}$$
$$= \frac{2(a+b+c)}{(a+b)(b+c)(c+a)}$$
$$= \frac{2 \times 0}{(a+b)(b+c)(c+a)}$$
= 0

S5. Ans.(c)
Sol.
$$x = \frac{\sqrt{5}+1}{\sqrt{5}-1}$$
$$y = \frac{\sqrt{5}-1}{\sqrt{5}+1}$$
$$xy = 1$$
$$x+y = \frac{5+1+2\sqrt{5}+5+1-2\sqrt{5}}{5-1}$$
$$x+y = 3$$
$$x^2+y^2+2xy = 9$$
$$x-y = 1$$
$$\therefore x^2+y^2 = 7$$
$$\frac{x^2+xy+y^2}{x^2-xy+y^2} = \frac{7+1}{7-1}$$
$$= \frac{8}{6}$$
$$= \frac{4}{3}$$

S6. Ans.(c)

Sol.

$\left(a+\frac{1}{a}\right)^2 = 3$

$a+\frac{1}{a} = \sqrt{3}$

if $a+\frac{1}{a} = \sqrt{3}$

Then

$a^3+\frac{1}{a^3} = 0$

$a^6+1 = 0$

$a^{18}+a^{12}+a^6+1$

$= a^{12}(a^6+1)+a^6+1$

$= a^{12}\times 0+0$

$= 0$

S7. Ans.(d)

Sol.

$a^2 + b^2 + c^2 - ab - bc - ac$

$= \frac{1}{2}[(a-b)^2+(b-c)^2+(c-a)^2]$

a = 113, b = 115, c = 117

$\Rightarrow \frac{1}{2}[(113-115)^2+(115-117)^2+(117-113)^2]$

$\Rightarrow \frac{1}{2}[(-2)^2+(-2)^2+(4)^2]$

$= \frac{1}{2}(4+4+16)$

= 24 = 12

S8. Ans.(d)

Sol.

a + b = 1

Cubing both sides

$a^3+b^3+3ab(a+b) = 1$

$a^3+b^3+3ab = 1$

$-a^3-b^3 = 3ab-1$

$a^4+b^4-a^3-b^3-2a^2b^2+ab$

$= a^4+b^4+3ab-1-2a^2b^2+ab$

$= (a^4+b^4-2a^2b^2)+4ab-1$

$= (a^2-b^2)^2+4ab-1$

$= [(a+b)(a-b)]^2+4ab-1$

$= (a-b)^2+4ab-1$

$= a^2+b^2-2ab+4ab-1$

$= a^2+b^2+2ab-1$

$= (a+b)^2-1$

= 1 – 1 = 0

S9. Ans.(a)

Sol.

$x^2 + y^2 + 6x + 5 = 4x - 4y$
$x^2 + y^2 + 2x + 4y + 5 = 0$
$x^2 + y^2 + 2x + 4y + 1 + 4 = 0$
$x^2 + 2x + 1 + y^2 + 4y + 4 = 0$
$(x+1)^2 + (y+2)^2 = 0$
$x + 1 = 0 \mid y + 2 = 0$
$x = -1 \mid y = -2$
$x - y = -1 + 2$
$= 1$

S10. Ans.(c)

Sol.

$2a^3 + 2b^3 + 2c^3 - 6abc$
$\Rightarrow 2(a^3 + b^3 + c^3 - 3abc)$
$= 2 \times \frac{1}{2}(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2]$
$= (299 + 298 + 297)[(1)^2 + (1)^2 + (2)^2]$
$= 894 \times 6 = 5364$

S11. Ans.(d)

Sol.

$AB = AC$
$if\ \angle A = 60°$
Then, $a = b = c$
$If\ \angle A = 90°$
$a = \sqrt{2}b, \sqrt{2}c$
$c < a < c\sqrt{2}$

S12. Ans.(a)

Sol.

$\angle TCD = \frac{\pi}{4} = 45°$
$\therefore \angle OCB = 45° = \angle BOC$
$OB = BC = \frac{CD}{2} = 5\ cm$
Radius of circle = OC $= 5\sqrt{2}$
$AC = \sqrt{AB^2 + BC^2}$
$AC = \sqrt{(5\sqrt{2}+5)^2 + 5^2} = \sqrt{50 + 25 + 50\sqrt{2} + 25}$
$AC = \sqrt{100 + 50\sqrt{2}}$
$AC = 5\sqrt{4 + 2\sqrt{2}}$
Required perimeter $= AC + OC + OA$
$= 5\sqrt{4+2\sqrt{2}} + 5\sqrt{2} + 5\sqrt{2}$
$= 10\sqrt{2} + 5\sqrt{4+2\sqrt{2}}$
≈ 27.20

S13. Ans.(a)

Sol.

$\angle ADB = 120°$

$\therefore \angle ACB = 60°$ ($\because$ ACBD is a cyclic quadrilateral)

In ΔABC,

$AC = 12$ cm, $\angle ACB = 60°$

We can get $AB = 6\sqrt{3}, BC = 6$

Area of triangle $= \frac{1}{2} \times 6\sqrt{3} \times 6 = 18\sqrt{3}$ cm^2

S14. Ans.(a)

Sol.

AD : CD = 1 : 3

Area of $\Delta ABD = \frac{1}{2} AD \times Height$

Area of $\Delta BDC = \frac{1}{2} CD \times Height$

$\frac{Area\ of\ \Delta ABD}{Area\ of\ \Delta BDC} = \frac{AD}{CD} = \frac{1}{3} \Rightarrow \frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta ABD} = \frac{4}{1} \ldots\ldots\ldots\ldots\ldots (i)$

In same way, $\frac{Area\ of\ \Delta ABE}{Area\ of\ \Delta AED} = \frac{1}{2}$

$\Rightarrow \frac{Area\ of\ \Delta AED + Area\ of\ \Delta ABE}{Area\ of\ \Delta ABF} = \frac{3}{1}$

$\Rightarrow \frac{Area\ of\ \Delta ABD}{Area\ of\ \Delta ABE} = \frac{3}{1} \ldots\ldots\ldots\ldots\ldots (ii)$

From (i) & (ii) we can get

$\frac{Area\ of\ \Delta ABE}{Area\ of\ \Delta ABC} = \frac{1}{12}$

S15. Ans.(a)

Sol.

Δ ADE ~Δ ABC and Δ ~ Δ EFC ~ Δ ABC

$\frac{AE}{AC} = \frac{DE}{BC}$

And

$\frac{EC}{AC} = \frac{EF}{AB}$

$\therefore \frac{AE}{EC} = \frac{AB}{BC} = \frac{7.5}{10.5} = \frac{5}{7}$

$AB^2 + BC^2 = AC^2$

$(5x)^2 + (7x)^2 = 18^2$

$x^2 = \frac{324}{74}$

Area of Δ ABC $= \frac{1}{2} \times 5x \times 7x$

$= \frac{1}{2} \times 35 \times \frac{324}{74}$

$= 76.62\ cm^2$

S16. Ans.(b)

Sol.

$\angle ACB = \angle ADO = 90°$

Δ ADO ~ ΔACB

AO = OB (Radius of larger circle)

$\frac{AO}{AB} = \frac{AD}{AC}$

$\frac{AD}{AC} = \frac{1}{2}$

$\therefore AD = CD = 6\ cm$

S17. Ans.(a)

Sol.

∠CAD = 60°

∴ ∠DEC = 120° (∵ ACED is a cyclic quadrilateral)

∠DEB = 60°

∴ ∠BDE = 90°

⇒ ΔBDE is a right angle triangle

BD = 6 cm

We can get, BE = $4\sqrt{3}, and\ DE = 2\sqrt{3}$

∠ACB = 90° (∵ ∠ADE = 90°)

∴ Δ ABC is a right angle triangle

BC = CE + BE

BC = $5\sqrt{3} + 4\sqrt{3}$

$BC = 9\sqrt{3}\ cm$

∠CAB = 60° and ΔABC is a right angle triangle we can get

AC = 9, AB = 18

$\frac{AC}{AD} = \frac{9}{18-6} = \frac{3}{4}$

S18. Ans.(c)

Sol.

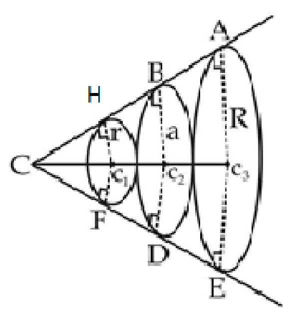
$x^2 + y^2 + z^2 = xy + yz + zx$

Equation will satisfy when x = y = z

∴ triangle is equilateral triangle

S19. Ans.(b)

Sol.



Let radius of smaller circle = r, and larger circle's radius = R

Radius of middle circle =a

∠ACE = 90°

$\angle AC_3E = 90°$

□ $ACEC_3$ *Will be a square*

In same way

□ CBC_2D *is a square*, CHC_1F *is also a square*

$CC_1 = r\sqrt{2}$

$CC_2 = a\sqrt{2} = a + r + r\sqrt{2}$

$\Rightarrow \frac{r}{a} = \frac{\sqrt{2}-1}{\sqrt{2}+1}$

$CC_3 = R\sqrt{2} = R + 2a + r + r\sqrt{2}$

$\frac{r}{R} = \frac{3-2\sqrt{2}}{3+2\sqrt{2}} = 17 - 12\sqrt{2}$

$or \frac{R}{r} = \frac{1}{17-12\sqrt{2}}$

S20. Ans.(c)

Sol.

Δ ADE ~ ΔABC

$\frac{DE}{BC} = \frac{65}{100}$

$\frac{DE}{BC} = \frac{13}{20}$

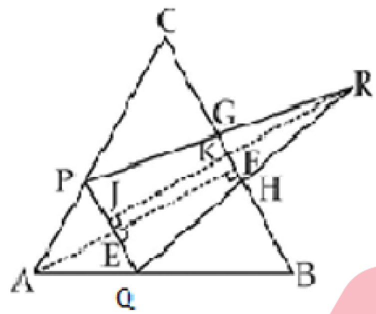
$\frac{Area\ of\ \Delta ADE}{Area\ of\ \Delta ABC} = \frac{DE^2}{BC^2} = \frac{169}{400}$

$Area\ of\ \Delta ADE = \frac{169}{400} \times 68\ cm^2$

$Area\ of\ \Delta ADE = 28.73\ cm^2$

S21. Ans.(a)

Sol.



P is mid point of AC

Q is mid point of AB

$\frac{AP}{PC} = \frac{AQ}{QB}$ $and\ AP = PC, AQ = BQ$

$\therefore \Delta AQP \sim \Delta\ ABC$

PQ || BC

$PQ = \frac{1}{2}BC \Rightarrow BC = 2PQ$

$AE = \frac{1}{2}AF \Rightarrow AF = 2AF$

Again Δ RGH ~ Δ RPQ

PQ = 2GH, RJ = 2RK, EF = JK

But since EF = AE = JK = RK

RJ = RK + JK and AF = AE + EF

⇒ RJ = AF = K(say)

$\frac{Area\ of\ \Delta PQR}{Area\ of\ \Delta ABC} = \frac{\frac{1}{2}\times PQ\times k}{\frac{1}{2}\times BC\times k} = \frac{PQ}{BC} = \frac{1}{2}$

S22. Ans.(b)

Sol.

Area of triangle ABC $= \frac{1}{2}b \times c \times sin\angle BAC$

$= \frac{1}{2}b \times c\ sin120°$

$= \frac{\sqrt{3}}{4}bc$

Also area of ΔABC $= Area\ of\ \Delta BAD + Area\ of\ \Delta CAD$

$\frac{\sqrt{3}}{4}bc = \frac{1}{2}c \times AD\ sin60° + \frac{1}{2} \times b\ AD\ sin60°$

$\frac{\sqrt{3}}{4}bc = \frac{\sqrt{3}}{4}AD\ (b + c)$

$AD = \frac{bc}{b+c}$

S23. Ans.(a)

Sol.

BC = OB (Given)

∴ $\angle BOC = \angle BCO = x°$

$\angle ABO = \angle BOC + \angle BCO$

$\angle ABO = 2x°$

$\angle ABO = \angle OAB = 2x°$

∴ $\angle AOB = 180° - (\angle OAB + \angle ABO)$ (∵ OAB is a triangle)

$\angle AOB = 180° - 4x$

$\angle AOD = 180° - (\angle AOB + \angle BOC)$

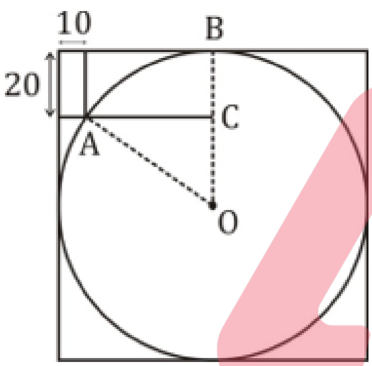
$\angle AOD = y = 180° - (180 - 4x + x)$

$\Rightarrow y = 3x$

Hence $k = 3$

S24. Ans.(c)

Sol.



OA = OB = r (radius)

OC = (r – 10)

AC = (r – 20)

$OC^2 + AC^2 = OA^2$

$(r-10)^2 + (r-20)^2 = r^2$

$r^2 - 60r + 500 = 0$

$r = 50, 10$

Out r cannot be 10

∵ In case of r = 10, B and will coincide.

∴ r = 50 cm

S25. Ans.(c)

Sol.

PQ ∥ AC ⇒ ΔACB ~ ΔPQB

∴ AP : PB = CQ : BQ = 4 : 3

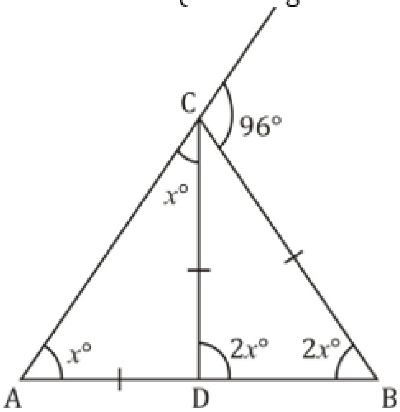
and QD ∥ CP ⇒ ΔCPB ~ ΔQDB

∴ CQ : BQ = PD : BD = 4 : 3

AP : PB = 4 : 3 and PD : BD = 4 : 3

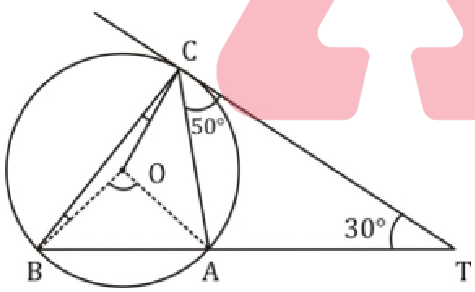
⇒ AP : PD = 7 : 3

S26. Ans.(c)
Sol.
AD = CD (given)
∴ ∠CAD = ∠ACD = $x°$ (say)
∴ ∠BDC = $2x°$ (outerangle of triangle)



∵ CD = BC
∴ ∠BDC = ∠DBC = $2x°$
Now, In Δ ABC,
∠CAB + ∠ABC = ∠BCE
$x° + 2x° = 96°$
$x = 32°$
∠DBC = 64°

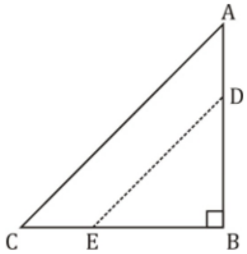
S27. Ans.(a)
Sol.



∠ACT = 50° (given)
And
OC ⊥ CT
∴ ∠OCT = 90°
∠ACO = 90° - 50°
∠ACO = 40°
And
∠CAT = 180° - (50° + 30°)
∠CAT= 100°
∴ ∠CAB = 80°
⇒ ∠BOC = 2 ∠CAB = 160°
⇒ ∠OBC = ∠OCB = 10°
Now we can get
∠ACB = ∠ACO + ∠OCB
∠ACB = 40° + 10°
∠ACB = 50°
and ∠BOA = 2∠ACB
∴ ∠BOA = 100°

S28. Ans.(d)

Sol.



DE ∥ AC

Area of ΔABC = 34 inch²

$DE = \frac{(100-35)}{100} AB$

$\frac{DE}{AC} = \frac{13}{20}$

ΔABC ~ ΔDBE (∵ AC ∥ DE)

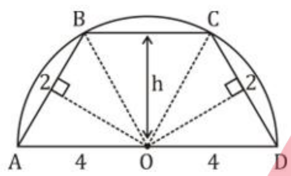
$\Rightarrow \frac{DE}{AC} = \frac{BD}{AB} = \frac{BE}{BC} = \frac{13}{20}$

$\Rightarrow \frac{area\ of\ \Delta DBE}{area\ of\ \Delta ABC} = \left(\frac{DE}{AC}\right)^2 = \frac{169}{400}$

$Area\ of\ \Delta DBE = \frac{169}{400} \times 34 = 14.365\ inch^2$

S29. Ans.(b)

Sol.



Area of ABCD $= \frac{1}{2}(8 + BC)h$

$\frac{1}{2}(8 + BC)h = 2\sqrt{15} + \frac{1}{2}BC \times h$

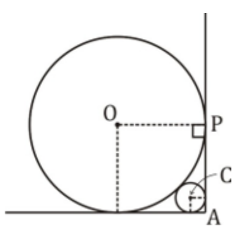
$\Rightarrow h = \frac{\sqrt{15}}{2}$

$\frac{BC}{2} = \sqrt{16 - \frac{15}{4}}$

⇒ BC = 7 cm

S30. Ans.(d)

Sol.



Let 'O' is centre of bigger circle.

And 'c' is centre of smaller circle.

and 'r' is radius of smaller circle.

OA = OP$\sqrt{2}$

OA = $2\sqrt{2}$

OA = radius of bigger circle + radius of smaller circle + AC

$2\sqrt{2} = 2 + r + r\sqrt{2}$

$r = \frac{2(\sqrt{2}-1)}{(\sqrt{2}+1)}$

$r = 2(2 + 1 - 2\sqrt{2}) = (6 - 4\sqrt{2})$