Quant Mega Quiz for SSC Tier – 1 (Solutions)

S1. Ans.(b)

Sol.

$\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=3$

➔ a = 3b; c = 3d; e = 3f

So, $\frac{2a^2+3c^2+4e^2}{2b^2+3d^2+4f^2}=\frac{2*9b^2+3*9d^2+4*9f^2}{2b^2+3d^2+4f^2}$

➔ $\frac{9(2b^2+3d^2+4f^2)}{2b^2+3d^2+4f^2}=9$

S2. Ans.(c)

Sol.

$\frac{x}{2x^2+5x+2}=\frac{1}{6}$

➔ $2x^2+5x+2=6x$

➔ $2x^2+2=6x-5x=x$

On dividing by 2,

➔ $x^2+1=\frac{x}{2}$

On dividing by x,

➔ $x+\frac{1}{x}=\frac{1}{2}$

S3. Ans.(a)

Sol.

$a^2+b^2+c^2=2a-2b-2c-3$

➔ $a^2-2a+b^2+2b+c^2+2c+1+1+1=0$

➔ $(a^2-2a+1)+(b^2+2b+1)+(c^2+2c+1)=0$

➔ $(a-1)^2+(b+1)^2+(c+1)^2=0$

➔ $a-1=0 \Rightarrow a=1$

➔ $b+1=0 \Rightarrow b=-1$

And $c+1=0 \Rightarrow c=-1$

➔ $2a-3b+4c$

$=2+3-4=1$

S4. Ans.(b)

Sol.

$a = \frac{\sqrt{5}+1}{\sqrt{5}-1} = \frac{\sqrt{5}+1}{\sqrt{5}-1} * \frac{\sqrt{5}+1}{\sqrt{5}+1}$

$\rightarrow \frac{(\sqrt{5}+1)^2}{5-1} = \frac{5+1+2\sqrt{5}}{4}$

$\rightarrow \frac{3+\sqrt{5}}{2}$

Similarly, $b = \frac{3-\sqrt{5}}{2}$

So, $a + b = \frac{3+\sqrt{5}}{2} + \frac{3-\sqrt{5}}{2} = 3$

And $ab = \frac{\sqrt{5}+1}{\sqrt{5}-1} * \frac{\sqrt{5}-1}{\sqrt{5}+1} = 1$

So, Expression $\frac{a^2+ab+b^2}{a^2-ab+b^2} = \frac{(a+b)^2-ab}{(a+b)^2-3ab}$

$\rightarrow \frac{9-1}{9-3} = \frac{8}{6} = \frac{4}{3}$

S5. Ans.(d)

Sol.

$x + \frac{1}{x} = 99$

So, $\frac{100x}{2x^2+102x+2} = \frac{100x}{2x^2+2+102x}$

On dividing by x,

$\rightarrow \frac{100}{2x + \frac{2}{x} + 102}$

$\rightarrow \frac{100}{2(x + \frac{1}{x}) + 102}$

$\rightarrow \frac{100}{2 * 99 + 102} = \frac{100}{300} = \frac{1}{3}$

S6. Ans.(c)

Sol.

$x = 3 + 2\sqrt{2}$

$\rightarrow \frac{1}{x} = \frac{1}{3+2\sqrt{2}} * \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$

$\rightarrow \frac{3-2\sqrt{2}}{9-8} = 3 - 2\sqrt{2}$

So, $(\sqrt{x} - \frac{1}{\sqrt{x}})^2 = x + \frac{1}{x} - 2$

$\rightarrow 3 + 2\sqrt{2} + 3 - 2\sqrt{2} - 2 = 4$

So, $(\sqrt{x} - \frac{1}{\sqrt{x}}) = \sqrt{4} = 2$

S7. Ans.(d)

Sol.

$\frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy}$

➔ $\frac{x^3+y^3+z^3}{xyz}$

➔ $\frac{3xyz}{xyz}$ = 3 [Since x + y + z = 0 then $x^3 + y^3 + z^3 = 3xyz$]

Or you can directly put x = 1, y = 1 and z = -2

S8. Ans.(a)

Sol.

$3x - 2 = \frac{3}{x}$

➔ $3x - \frac{3}{x} = 2$

➔ $x - \frac{1}{x} = \frac{2}{3}$

On squaring both sides

➔ $(x - \frac{1}{x})^2 = \frac{4}{9}$

➔ $x^2 + \frac{1}{x^2} - 2 = \frac{4}{9}$

➔ $x^2 + \frac{1}{x^2} = \frac{4}{9} + 2 = \frac{22}{9} = 2\frac{4}{9}$

S9. Ans.(b)

Sol.

x + y + z = 13

$x^2 + y^2 + z^2 = 69$

➔ $(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$

➔ $(13)^2 = 69 + 2(xy + yz + zx)$

➔ $2(xy + yz + zx) = 169 - 69$

➔ $xy + yz + zx = \frac{100}{2} = 50$

S10. Ans.(c)

Sol.

$x = \frac{1}{2+\sqrt{3}}$

➔ $\frac{1}{2+\sqrt{3}} * \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{4-3} = 2 - \sqrt{3}$

Similarly, $y = \frac{1}{2-\sqrt{3}} = 2 + \sqrt{3}$

So, $x + y = 2 - \sqrt{3} + 2 + \sqrt{3} = 4$

$Xy = (2 - \sqrt{3})(2 + \sqrt{3}) = 4 - 3 = 1$

So, $8xy\ (x^2 + y^2)$

$= 8xy\ [(x + y)^2 - 2xy]$

$= 8 * 1\ (4^2 - 2 * 1)$

$= 8\ (16 - 2) = 8 \times 14 = 112$

S11. Ans.(a)

Sol.

$2x + \frac{1}{4x} = 1$

On dividing by 2, we get

➔ $x + \frac{1}{8x} = \frac{1}{2}$

On squaring both sides, we get

➔ $(x + \frac{1}{8x})^2 = \frac{1}{4}$

➔ $x^2 + \frac{1}{64x^2} + 2 * x * \frac{1}{8x} = \frac{1}{4}$

➔ $x^2 + \frac{1}{64x^2} + \frac{1}{4} = \frac{1}{4}$

➔ $x^2 + \frac{1}{64x^2} = \frac{1}{4} - \frac{1}{4} = 0$

S12. Ans.(d)

Sol.

p = 99 (Given)

Expression = $p(p^2 + 3p + 3)$

$= p^3 + 3p^2 + 3p$

$= p^3 + 3p^2 + 3p + 1 - 1$

$= (p + 1)^3 - 1$

$= (98 + 1)^3 - 1 = (99)^3 - 1$

$= 970299 - 1$

$= 970298$

S13. Ans.(c)

Sol.

$x = \sqrt{3} + \sqrt{2}$

So, $\frac{1}{x} = \sqrt{3} - \sqrt{2}$ (by rationalization)

So, $x + \frac{1}{x} = \sqrt{3} + \sqrt{2} + \sqrt{3} - \sqrt{2} = 2\sqrt{3}$

Now, $x^3 + \frac{1}{x^3} = (x + \frac{1}{x})^3 - 3 * x * \frac{1}{x}(x + \frac{1}{x})$

➔ $(2\sqrt{3})^3 - 3(2\sqrt{3})$

➔ $24\sqrt{3} - 6\sqrt{3} = 18\sqrt{3}$

S14. Ans.(d)

Sol.

Given $(x + \frac{1}{x})^2 = 3$

➔ $(x + \frac{1}{x}) = \sqrt{3}$

On cubing both sides we get,

➔ $x^6 + 1 = 0$ or $x^6 = -1$

Now, $x^{234} + x^{228} + x^{206} + x^{200} + x^{90} + x^{84} + x^{18} + x^{12} + x^6 + 1$

➔ $x^{228}(x^6 + 1) + x^{200}(x^6 + 1) + x^{84}(x^6 + 1) + x^{12}(x^6 + 1) + x^6 + 1$

➔ $x^{228}(-1 + 1) + x^{200}(-1 + 1) + x^{84}(-1 + 1) + x^{12}(-1 + 1) - 1 + 1$

➔ 0

S15. Ans.(c)

Sol. x + y + z = 6

On squaring,

$x^2 + y^2 + z^2 + 2xy + 2zy + 2zx = 36$

➔ $20 + 2\,(xy + yz + zx) = 36$

➔ $xy + yz + zx = 8$

So, $x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z)\,(x^2 + y^2 + z^2 - xy - yz - zx)$

$= (x + y + z)\,[x^2 + y^2 + z^2 - (xy + yz + zx)]$

$= 6\,(20 - 8)$

$= 72$

S16. Ans.(d)

Sol.

$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)\,[(x - y)^2 + (y - z)^2 + (z - x)^2]$

➔ $\frac{1}{2}(333 + 334 + 335)\,[(333 - 334)^2 + (334 - 335)^2 + (335 - 333)^2]$

➔ $\frac{1}{2}(1002)\,[(1 + 1 + 4]$

➔ $\frac{1}{2} * 1002 * 6$

➔ 3006

S17. Ans.(c)

Sol.

$x + \frac{1}{x} = 2$

➔ $x^2 + 1 = 2x$

➔ $x^2 - 2x + 1 = 0$

➔ $(x - 1)^2 = 0 \Rightarrow x = 1$

So, $x^{1331} + \frac{1}{x^{1332}} = 1 + 1 = 2$

S18. Ans.(a)

Sol.

$a + b + c = 1$

$ab + bc + ca = -1$

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

➔ $1 = a^2 + b^2 + c^2 + 2\,(-1)$

➔ $a^2 + b^2 + c^2 = 3$

$a^3 + b^3 + b^3 - 3abc = (a + b + c)\,(a^2 + b^2 + c^2 - ab - bc - ca)$

$= 1\,(3 + 1)$

$= 4$

$a^3 + b^3 + b^3 = 3abc + 4$

➔ $-3 + 4 = 1$ [∵ abc = -1]

S19. Ans.(c)

Sol.

$xy\,(x+y) = m$ (Given)

➔ $x^3 + y^3 + 3m$

➔ $x^3 + y^3 + 3xy\,(x+y)$

➔ $(x+y)^3$

➔ $(\frac{m}{xy})^3 = \frac{m^3}{x^3\,y^3}$

S20. Ans.(b)

Sol.

$\frac{a}{b} + \frac{b}{a} = 1$

➔ $\frac{a^2+b^2}{ab} = 1$

➔ $a^2 + b^2 = ab$

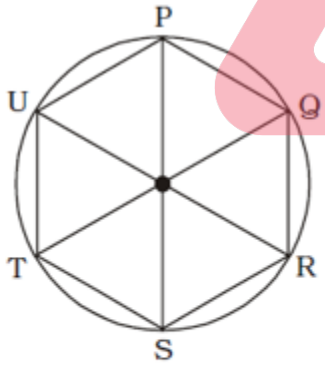
➔ $a^2 - ab + b^2 = 0$

➔ $\therefore a^3 + b^3 = (a+b)\,(a^2 - ab + b^2)$

➔ $a^3 + b^3 = (a+b) * 0 = 0$

S21. Ans.(b)

Sol.



Each triangle is equilateral.

$\therefore \angle P + \angle R + \angle T$

$= 2\,(60° + 60° + 60°)$

$= 2 \times 180° = 360°$

S22. Ans.(d)

Sol.

Sum of two supplementary angles = 180°

$\therefore 5y + 62° + 22° + y = 180°$

➔ $6y + 84° = 180°$

➔ $6y = 180° - 84° = 96°$

➔ $\therefore y = \frac{96}{6} = 16°$

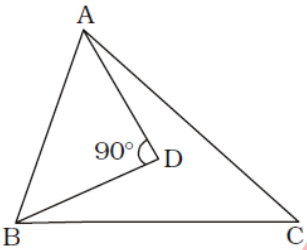
S23. Ans.(c)
Sol.
The opposite angles of a parallelogram are equal.
$3x + 6x + 3x + 6x = 360°$
➔ $8x = 360°$
➔ $x = 20°$

One angle = $3x = 3 \times 20 = 60°$
Second angle = $6x = 6 \times 20 = 120°$
$\therefore 120° + 60° = 180°$

Note: A Rhombus is always a parallelogram

but all parallelogram are not rhombus.

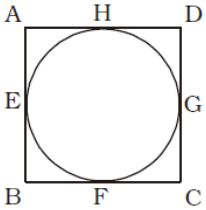
S24. Ans.(a)
Sol.



$\angle A = \angle B = \angle C = 60°$
BC = AB = AC = 13 cm
BD = 2 cm

In DABD,
$AB^2 = BD^2 + AD^2$
➔ $(\sqrt{13})^2 = (2)^2 + AD^2$
➔ $AD^2 = 13 - 4 = 9$
➔ AD = 3 cm

S25. Ans.(d)
Sol.

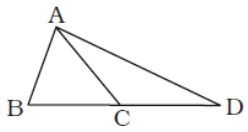


AE = AH
BE = BF
GC = FC
GD = HD

➔ AE + BE + GC + GD = AH + BF + FC + HD
➔ AB + CD = AD + BC
➔ 6 + 3 = AD + 7.5
➔ AD = 9 – 7.5 = 1.5 cm

S26. Ans.(b)

Sol.



∠ACB = 80°

∴ ACD = 180° – 80° = 100°

∠CAD = ∠CDA [CD = AC]

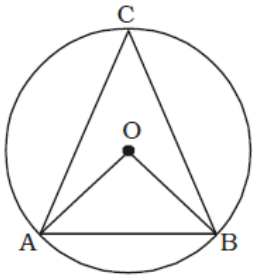
$= \frac{80}{2} = 40°$

∠BAC = 111° – 40° = 71°

∠ABC = 180° – 71° – 80° = 29°

S27. Ans.(c)

Sol.



OA = OB = AB

∴ OAB is an equilateral triangle.

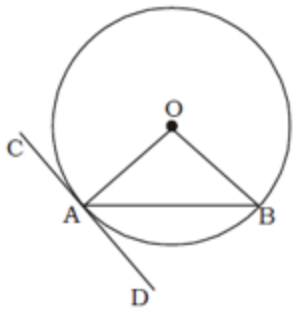
∴ ∠AOB = 60°

∴ ACB = $\frac{60}{2}$ = 30°

Angle subtended at the center by an arc is twice to that at the circumference.

S28. Ans.(b)

Sol.



Tangent ==> CAD

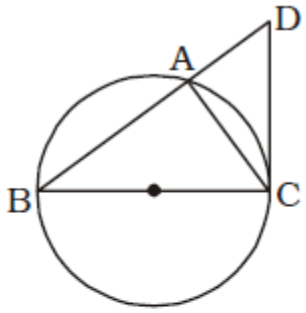
OA ⊥ CD

➔ OAD = 90°

∠BAD = θ

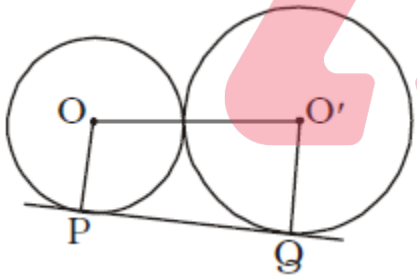
∴ ∠OAB = 90° – θ = ∠OBA

S29. Ans.(a)
Sol.



The angle at the circumference of a semi-circle is a right angle.
∴ ∠BAC = 90°
∠ABC = 36°
∴ ACB = 90° – 36° = 54°
∠BCD = 90°
∴ ACD = 90° – 54° = 36°
∠DAC = 90°
∴ ADC = 90° – 36° = 54°

S30. Ans.(c)
Sol.



PQ = $\sqrt{(OO')^2 - (r_2 - r_1)^2}$

➔ $\sqrt{(13)^2 - (8-3)^2}$

➔ $\sqrt{169 - 25}$

➔ $\sqrt{144}$ = 12 cm